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LETTER TO THE EDITOR

Heat conduction in relativistic extended thermodynamics

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Abstract. We outline an approach to irreversible thermodynamics leading to a covariant evolution equation for the four-flux of heat in a relativistic non-viscous simple fluid. This relation results in a non-trivial generalisation of the classical case.

Starting from Muller's (1967) Newtonian thermodynamics approach, Israel (1976) and Israel and Stewart (1979) have obtained a covariant and causal theory describing the dissipative processes in a fluid. The main point of that theory is the generalisation of the entropy four-flux by the inclusion of quadratic terms in heat four-flux and viscosity, leading to a finite speed for the propagation of both dissipative effects.

Our aim in this Letter is to derive a causal evolution equation for heat conduction in a relativistic non-viscous simple fluid from a phenomenological theory: that of extended irreversible thermodynamics, such as formulated by Lebon *et al* (1980) and Jou *et al* (1979). This description is proved to be more general than Muller's, being at the same time consistent with Grad's kinetic approach. In the non-relativistic formulation, this description assumes that the state of the system is specified by the following independent variables: internal energy, density, velocity and the heat flux vector $\boldsymbol{q}$ whose evolution has to be determined. Since the evolution of the first three variables is given by the classical laws of mass, momentum and energy balance, the main problem of thermodynamics is to obtain the evolution of heat flux.

Usually, the symmetric momentum–energy tensor of our system is considered to be decomposed into two mutually orthogonal directions according to

$$T^{\mu\nu} = \epsilon u^{\mu}u^{\nu} + 2c^{-1}q^{(\mu}u^{\nu)} + p\Delta^{\mu\nu}. \tag{1}$$

Here ϵ, u^{μ}, q^{μ} and p are respectively the internal specific energy, the unit four-velocity, the heat four-flux and the hydrostatic scalar pressure, while $\Delta^{\mu\nu}$ is the spatial projector $g^{\mu\nu} + u^{\mu}u^{\nu}$. The energy conservation equation $\partial_{\mu}(-u_{\nu}T^{\mu\nu}) = 0$ gives rise to the relation (see Eckart (1940))

$$\rho(\mathrm{D}\epsilon + p\mathrm{D}v) + c^{-1}(\partial_{\mu}q^{\mu} + q^{\mu}\mathrm{D}u_{\mu}) = 0 \tag{2}$$

where ρ denotes mass density, v is the specific volume and D stands for the invariant derivative $u^{\mu}\partial_{\mu}$ along the fluid flux.

In extended irreversible thermodynamics, in addition to the classical variables, the dissipative fluxes enter into the specific entropy function as well as in the entropy flux. In our case,

$$s = s(\epsilon, v, q^{\mu}) \qquad I_s^{\mu} = I_s^{\mu}(\epsilon, v, q^{\mu}). \tag{3}$$

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At this point, we establish the following equations of state:

$$\left.\frac{\partial s}{\partial \epsilon}\right|_{v,q^{\mu}} = T^{-1}; \qquad \left.\frac{\partial s}{\partial v}\right|_{\epsilon,q^{\mu}} = pT^{-1}; \qquad \left.\frac{\partial s}{\partial q^{\mu}}\right|_{\epsilon,v} = (\rho T)^{-1}A^{\mu}. \tag{4}$$

The two first define, in analogy with the classical theory, the absolute temperature T and the thermodynamic pressure p respectively, whereas the third one, which vanishes identically at equilibrium, defines a new state parameter. The most general expression of this four-vector A^{μ} in terms of the thermodynamic variables up to first order reads

$$A^{\mu} = \alpha(\epsilon, v)q^{\mu}, \tag{5}$$

where α denotes a parameter to be identified later.

We may express the entropy four-flux I_s^{μ} up to first order as

$$I_s^{\mu} = \beta(\epsilon, v)q^{\mu} \tag{6}$$

where β is another parameter which will also be determined below.

With (4) and (5), differentiation of s yields the covariant and generalised Gibbs equation

$$\mathrm{D}s = T^{-4}(\mathrm{D}\epsilon + p\mathrm{D}v + \alpha v q^{\mu}\mathrm{D}q_{\mu}). \tag{7}$$

Elimination of $\mathrm{D}\epsilon$ between (2) and (7) leads, after comparison with the standard entropy balance equation

$$\rho\mathrm{D}s + \partial_{\mu}I_s^{\mu} = \sigma, \tag{8}$$

to the following identifications:

$$\beta = 1/cT \tag{9}$$

and

$$\sigma = -\frac{1}{cT^2}q^{\mu}(\partial_{\mu}T + T\mathrm{D}u_{\mu} - c\alpha T\mathrm{D}q_{\mu}). \tag{10}$$

Since the entropy production σ must be a positive semidefinite quantity, the most simplifying assumption about the relation between the bracket in (10) and q^{μ} reads

$$q^{\mu} = -k\Delta^{\mu\nu}(\partial_{\nu}T + T\mathrm{D}u_{\nu} - c\alpha T\mathrm{D}q_{\nu}) \tag{11}$$

where $k(\geqslant 0)$ is the thermal conductivity of the fluid.

In order to obtain the evolution equation for q^{μ} we solve for $\mathrm{D}q^{\mu}$ in (11); this can be achieved by using both the definition of $\Delta^{\mu\nu}$ and the orthogonality relation $u^{\mu}q_{\mu} = 0$; then we find

$$\mathrm{D}q^{\mu} = (kc\alpha T)^{-1}[q^{\mu} + k\Delta^{\mu\nu}(\partial_{\nu}T + T\mathrm{D}u_{\nu}) + kc\alpha T u^{\mu}q_{\nu}\mathrm{D}u^{\nu}]. \tag{12}$$

This relation gives us the evolution of q^{μ} in the direction of u^{μ}, i.e. along the world line of fluid particles, and has two parts clearly different: one, $(kc\alpha T)^{-1}[q^{\mu} + k\Delta^{\mu\nu}(\partial_{\nu}T + T\mathrm{D}u_{\nu})]$, orthogonal to u_{μ}, and another, $u^{\mu}q_{\nu}\mathrm{D}u^{\nu}$, parallel to u^{μ}. This latter takes into account the contribution to $\mathrm{D}q^{\mu}$ due to work done by the heat four-flux passing through the accelerated fluid.

In order to determine the parameter α, we analyse the phenomenological relation (11), (or (12)) in the rest frame of the fluid. In it, we have $\Delta^{\mu\nu} = \mathrm{diag}(0, 1, 1, 1)$,

$D \to \partial/\partial ct$, $q^{\mu} \to (0, \boldsymbol{q})$, $Du_{\mu} = 0$, and consequently equation (11) (and also equation (12)) becomes the well-known Maxwell–Cattaneo equation

$$\boldsymbol{q} + \tau(\partial \boldsymbol{q}/\partial t) = -k\nabla T \tag{13}$$

where τ is the proper relaxation time of the process, provided that $\alpha = -\tau/kT$.

Assuming τ to be of the order of a molecular mean collision time, the resulting speed of heat pulse is not only lower than the speed of light but is moreover comparable with the speed of sound in the fluid under consideration (see Israel (1976)).

The covariant theory of non-equilibrium thermodynamics presented here is a non-trivial generalisation of the usual theory, as shown by the presence of the terms TDu_{ν} and $u^{\mu}q_{\nu}Du^{\nu}$ in (12).

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